Indian Statistical Institute, Bangalore

B. Math. Third Year

First Semester - Combinatorics

Mid-Semester Exam | Duration : 3 hours | Date : Sept 11, 2015

Remark: There are five questions. Each carries 20 marks. Full Marks: 100

1. Let X be a finite linear space in which any two lines intersect. Then show that X is a projective plane, possibly degenerate.

2. Let X be a $2-(\vartheta, k, \lambda)$ design such that $k(k-1) = \lambda(\vartheta - 1)$. Then show that the dual X^* is also a $2-(\vartheta, k, \lambda)$ design.

3. Prove that every oval in a finite projective plane of even order extends uniquely to a hyper oval. Is this true for odd order?

4. Let $\mathbb{F}$ be an arbitrary field. Consider the points e_1, e_2, e_3, e_4 of $PG(2, \mathbb{F})$ where $e_1 = [1:0:0], e_2 = [0:1:0], e_3 = [0:0:1]$ and $e_4 = [1:1:1]$. Let f be an automorphism of $PG(2, \mathbb{F})$ such that $f(e_i) = e_i, 1 \leq i \leq 4$. Then show that there is an automorphism α of $\mathbb{F}$ such that $f([x_1 : x_2 : x_3]) = [\alpha(x_1) : \alpha(x_2) : \alpha(x_3)]$ for all points $[x_1 : x_2 : x_3]$ of PG $(2, \mathbb{F})$.

5. If $\mathbb{F}$ is a field of odd characteristic then show that the dual of any conic of $PG(2, \mathbb{F})$ is a conic in the dual plane.